Chapter 6.

Quadratic equations.

Try to solve the following problems. You might try to set up an equation and then solve it. If you find that approach difficult an alternative approach would be to try a value, test to see if it works and, if it does not, adjust your values for a new trial, i.e. "trial and adjustment".

Situation One.

I think of a number, multiply it by itself, take away four times the number I first thought of and end up with an answer of 21. What was the number I first thought of?

Situation Two.

I think of a number, multiply it by itself, take away ten times the number I first thought of, add 21 and end up with zero. What was the number I first thought of?

Situation Three.

If an object is dropped, the distance, s metres, that it has fallen x seconds later is given by the rule:

$$s = \frac{49}{10}x^2$$

If an object is dropped from a bridge, how long does it take to reach the water, 65 metres below? (Give your answer to the nearest tenth of a second.)

Situation Four.

If an object is thrown vertically downwards with initial speed 5m/s the distance, s metres, that it has fallen x seconds later is given by the rule:

$$s = 5x + \frac{49}{10}x^2$$

If the object referred to in situation three were to be thrown vertically downwards with initial speed 5m/s rather than dropped, how long does it take to reach the water now? (Again give your answer to the nearest tenth of a second.)

How did you get on with the situations on the previous page?
Perhaps for situation one you used trial and adjustment as shown below.

Try $x = 1$	$(1)^2 - 4(1) = -3$,	not 21.	Thus $x \neq 1$.
Try $x = 4$	$(4)^2 - 4(4) = 0$,	not 21.	Thus $x \neq 4$.
Try $x = 6$	$(6)^2 - 4(6) = 12$,	not 21.	Thus $x \neq 6$.
Try $x = 7$	$(7)^2 - 4(7) = 21$,	as required.	Thus $x = 7$.

The number first thought of was 7.
However, did you also discover that the number first thought of could have been −3?:

$$\text{If } x = -3 \qquad (-3)^2 - 4(-3) = 9 + 12 = 21, \text{ as required.} \qquad \text{Thus } x = -3.$$

Alternatively, perhaps you set up an equation as shown below:

Let x be the number thought of:	x
Multiply it by itself:	x^2
Take away 4 times the number first thought of:	$x^2 - 4x$
End up with 21:	$x^2 - 4x = 21$

The Preliminary work section at the beginning of this text mentioned that an ability to solve readily factorisable quadratic equations is assumed so you may have solved this equation as shown below, and arrived at the two possible solutions directly.

Given	$x^2 - 4x = 21$		
i.e.	$x^2 - 4x - 21 = 0$		
Factorising	$(x - 7)(x + 3) = 0$		
Hence either	$x - 7 = 0$	or	$x + 3 = 0$
Giving solutions of	$x = 7$		$x = -3$

Similarly did you discover the two possible numbers for situation two? In that case the equation was $x^2 - 10x + 21 = 0$ and both 3 and 7 satisfy this equation.

Situation three involved solving the equation $65 = \frac{49}{10}x^2$

$$\text{i.e. } 650 = 49x^2$$

$$\text{giving } x^2 = \frac{650}{49}$$

Once again there are two solutions to this equation. They are, correct to one decimal place, 3·6 and −3·6. However, given the context of the question, x cannot be negative so there is only one realistic solution. Thus the object would take 3·6 seconds to hit the water.

Situation four involved solving the equation $65 = 5x + \frac{49}{10}x^2$

$$\text{i.e. } 0 = 4{\cdot}9x^2 + 5x - 65$$

Once again there are two solutions to this equation, $x = 3{\cdot}2$ and $x = -4{\cdot}2$, correct to one decimal place, but only the positive answer is realistic, given the context. Thus the object would take 3·2 seconds to hit the water.

All four of the situations involved solving equations that could be written in the form:

$$ax^2 + bx + c = 0.$$

In situation one	$x^2 - 4x - 21 = 0$	i.e.	a = 1,	b = −4,	c = −21.
In situation two	$x^2 - 10x + 21 = 0$	i.e.	a = 1,	b = −10,	c = 21.
In situation three	$49x^2 - 650 = 0$	i.e.	a = 49,	b = 0,	c = −650.
In situation four	$4{\cdot}9x^2 + 5x - 65 = 0$	i.e.	a = 4·9,	b = 5,	c = −65.

Equations that can be written in the form $\boxed{ax^2 + bx + c = 0,\ a \neq 0}$ are called **quadratic** equations.

☞ In the previous chapter we looked at the graphs of functions with a rule that could be written in the form $y = ax^2 + bx + c$, i.e. **quadratic functions**.
For each value of x the rule assigns one and only one value for y. Hence, a function.

☞ In this chapter we will consider ways of solving equations of the form
$ax^2 + bx + c = 0$, i.e. **quadratic equations**.
Solving the equation means finding the value(s) of x for which the equality is true.

☞ Again, to repeat what was said in the Preliminary work section, it is anticipated that the reader is already familiar with solving "readily factorisable quadratic equations". In case this assumed ability is a little "rusty" example 1 below, and the exercise that follows, provides practice.

☞ Solving a quadratic equation by factorising uses the fact that **if two numbers have a product of zero then at least one of the numbers must be zero**. Just pause a moment to check that you understand and agree with this fact.
Thus if $(x + 3)(x + 2) = 0$ it follows that either $(x + 3) = 0$ or $(x + 2) = 0$.

Example 1

Solve each of the following equations.

(a) $x^2 - 6x = 0$ (b) $x^2 - 2x - 3 = 0$ (c) $x^2 - 13x = 30$
(d) $x^2 = 15x$ (e) $(2x - 5)(x + 7) = 0$ (f) $x^2 = 16$
(g) $2x^2 + 7x - 15 = 0$

(a) Given: $x^2 - 6x = 0$
Factorising: $x(x - 6) = 0$
Hence either $x = 0$ or $x - 6 = 0$
$x = 0$ or $x = 6$

(b) Given: $x^2 - 2x - 3 = 0$
(To factorise $x^2 - 2x - 3$ we look for two numbers that add to give −2 and multiply to give −3. The numbers are −3 and +1.)
Factorising: $(x - 3)(x + 1) = 0$
Hence either $x - 3 = 0$ or $x + 1 = 0$
$x = 3$ or $x = -1$

(c) Given: $x^2 - 13x = 30$

$\therefore$ $x^2 - 13x - 30 = 0$

(To factorise $x^2 - 13x - 30$ we look for two numbers that add to give −13 and multiply to give −30. The numbers are −15 and +2.)

Factorising: $(x - 15)(x + 2) = 0$

Either $x - 15 = 0$ or $x + 2 = 0$

$x = 15$ or $x = -2$

(d) Given $x^2 = 15x$

$\therefore$ $x^2 - 15x = 0$

Factorising: $x(x - 15) = 0$

Either $x = 0$ or $x - 15 = 0$

$x = 0$ or $x = 15$

(e) Given $(2x - 5)(x + 7) = 0$ (Note that this is already in factorised form.)

Either $2x - 5 = 0$ or $x + 7 = 0$

$2x = 5$ or $x = -7$

$x = 2{\cdot}5$ or $x = -7$

(f) Given $x^2 = 16$

$x = -4$ or $+4$

written $x = \pm 4$

These answers could be obtained by factorising but it would be a longer process:

$$x^2 = 16$$
$$x^2 - 16 = 0$$
$$(x + 4)(x - 4) = 0$$

Either $x + 4 = 0$ or $x - 4 = 0$

$x = -4$ or $x = 4$

(g) Given: $2x^2 + 7x - 15 = 0$

(To factorise $ax^2 + bx + c$, when $a \neq 1$, we look for two numbers that add to give b and multiply to give ac. We then rewrite bx using these two numbers. Hence in this example we look for two numbers that add to give 7 and multiply to give −30. The numbers are 10 and −3.)

$$2x^2 + 7x - 15 = 0$$
$$2x^2 + 10x - 3x - 15 = 0$$
$$2x(x + 5) - 3(x + 5) = 0$$

$\therefore$ $(x + 5)(2x - 3) = 0$

Hence either $x + 5 = 0$ or $2x - 3 = 0$

$x = -5$ or $x = 1{\cdot}5$

Example 2

I think of a number, multiply it by itself, take away six times the number I first thought of and end up with fifty five. What was the number I first thought of?

Let the number first thought of be x.

The given information leads to the equation $x^2 - 6x = 55$

i.e. $x^2 - 6x - 55 = 0$

$(x - 11)(x + 5) = 0$

Either $x - 11 = 0$ or $x + 5 = 0$

$x = 11$ or $x = -5$

The number first thought of was either 11 or –5.

Exercise 6A

Solve each of the following equations. (Without the assistance of your calculator.)

1.	$(x + 5)(x - 3) = 0$	2.	$(x + 8)(x + 9) = 0$	3.	$(2x - 11)(x + 5) = 0$
4.	$x^2 = 25$	5.	$x^2 - 49 = 0$	6.	$2x^2 = 200$
7.	$x^2 + 9x + 20 = 0$	8.	$x^2 + x - 20 = 0$	9.	$x^2 - 9x + 20 = 0$
10.	$x^2 - x - 20 = 0$	11.	$x^2 + 2x - 35 = 0$	12.	$x^2 + 4x + 3 = 0$
13.	$x^2 + 7x + 6 = 0$	14.	$x^2 + 10x + 21 = 0$	15.	$x^2 + 8x + 15 = 0$
16.	$x^2 - 4x - 12 = 0$	17.	$x^2 - 4x - 5 = 0$	18.	$x^2 - 4x = 0$
19.	$x^2 + 5x - 14 = 0$	20.	$x^2 - 36 = 0$	21.	$x^2 + 6x + 9 = 0$
22.	$x^2 - 3x - 4 = 0$	23.	$x^2 - 8x + 16 = 0$	24.	$x^2 = 15 - 2x$
25.	$x^2 = 3x$	26.	$x^2 + 12 = 7x$	27.	$x^2 = 24 - 10x$
28.	$4x^2 - 9 = 0$	29.	$25x^2 - 1 = 0$	30.	$x^2 = 2x + 15$
31.	$x^2 + 9 = 6x$	32.	$x^2 = 5(2x - 5)$	33.	$2x^2 + 5x - 12 = 0$
34.	$3x^2 + 10x - 8 = 0$	35.	$2x^2 - 3x - 5 = 0$	36.	$5x^2 + 34x - 7 = 0$
37.	$2x^2 + x - 21 = 0$	38.	$6x^2 - 19x + 10 = 0$	39.	$10x^2 - 9x + 2 = 0$

40. When seven times a number is added to the square of the number the answer is 30. What is the number?

41. When ten times a number is added to the square of the number and twenty five is added to the total the final answer is zero. What is the number?

42. When an object is projected upwards from ground level, with initial speed 40 m/s, the height it is above ground t seconds later is h metres where h is given by:

$$h \approx 40t - 5t^2.$$

State the value of h when the object hits the ground again and find the value of t then.

43. If $s = ut + \frac{1}{2}at^2$ find the value of t given that $s = 10$, $u = 3$, $a = 2$ and $t \geq 0$.

44. If $w = kp^2 - 2cp$ find the value of p given that $w = 33$, $k = 1$ and $c = 4$.

What if the quadratic equation is not readily factorisable?

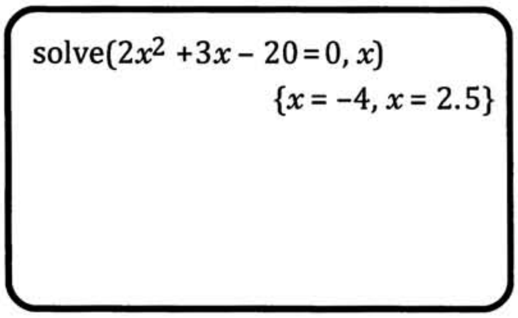
Given a quadratic equation $ax^2 + bx + c = 0$, when $ax^2 + bx + c$ is <u>not</u> readily factorised, we could

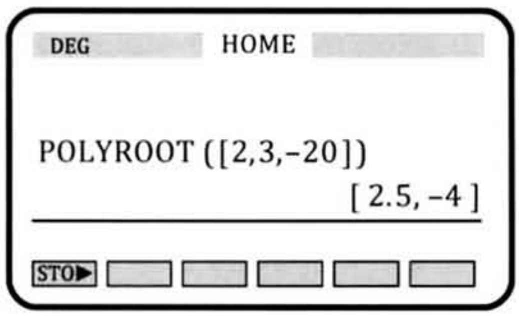
- use the built in facility of some calculators to solve equations.
- use our ability to sketch the graph of the function $y = ax^2 + bx + c$ and then, from the sketch, estimate the values of x for which $y = 0$.
- use the technique of completing the square from the previous chapter.
- use the technique of completing the square from the previous chapter to develop a formula for solving quadratic equations.

These approaches are discussed below

Using the built in facility of some calculators to solve equations.

Many calculators have built in routines for solving quadratic equations.
In some we can type in the equation as it is, see below left, and in others we input the values of a, b and c and the calculator provides the solutions, see below right.





Using a graphical approach.

The characteristic shape of quadratic functions encountered in the last chapter explains why an equation of the form

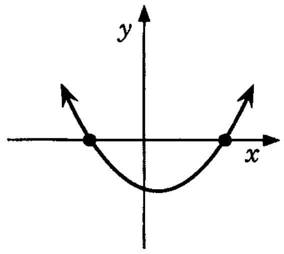
$$ax^2 + bx + c = 0$$

can have more than one solution (also called *roots*), as explained below.

- If the graph of $y = ax^2 + bx + c$ cuts the x-axis in two places then there must be two places on the graph where $y = 0$, and hence two places where $ax^2 + bx + c = 0$.
  In such situations the quadratic equation $ax^2 + bx + c = 0$ will have <u>two</u> solutions.

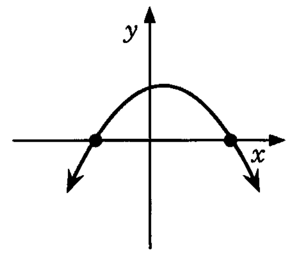
- If the graph of $y = ax^2 + bx + c$ just touches the x-axis in one place then there must be just one place where $ax^2 + bx + c = 0$.
  In such situations the quadratic equation $ax^2 + bx + c = 0$ will have <u>one</u> solution.

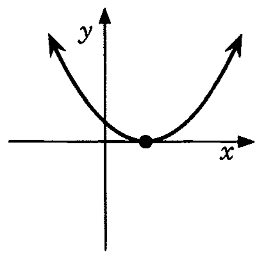
- If the graph of $y = ax^2 + bx + c$ neither cuts nor touches the x-axis then there are no places where $ax^2 + bx + c = 0$.
  In such situations the quadratic equation $ax^2 + bx + c = 0$ will have no solutions.



$y = ax^2 + bx + c$
cuts x-axis in two places.

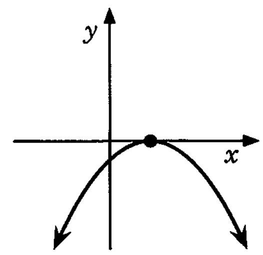
$ax^2 + bx + c = 0$
has two solutions.

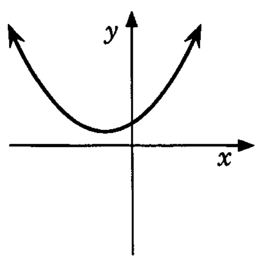




$y = ax^2 + bx + c$
touches x-axis in one place.

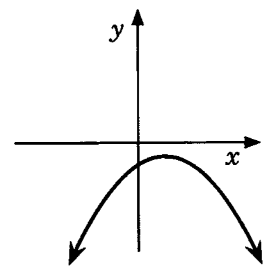
$ax^2 + bx + c = 0$
has one solution.





$y = ax^2 + bx + c$
neither cuts nor touches the x-axis.

$ax^2 + bx + c = 0$
has no solutions.



Thus to solve the equation $ax^2 + bx + c = 0$ we can sketch the graph of

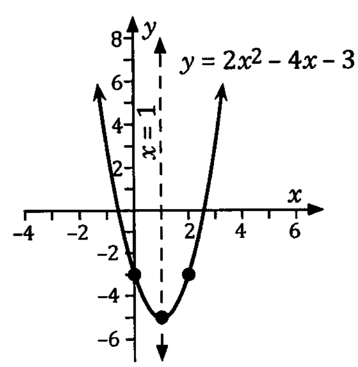
$$y = ax^2 + bx + c$$

and look at the x-coordinates of any points where the graph cuts the x-axis. At all such points y will equal zero and hence $\quad 0 = ax^2 + bx + c$

Example 3

Solve $2x^2 - 4x - 3 = 0$

Consider the function	$y = 2x^2 - 4x - 3$
Line of symmetry is	$x = 1$
Minimum point at	$(1, -5)$
Cuts y-axis at	$(0, -3)$



A sketch can then be made as shown on the right.

From this sketch: $\quad y = 0$ when $x \approx -0{\cdot}6$
and when $x \approx 2{\cdot}6$

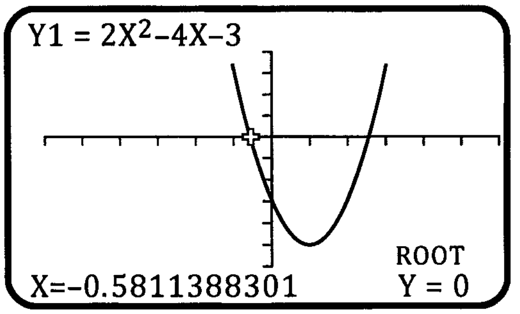
Hence $2x^2 - 4x - 3 = 0$ has solutions of $\quad x \approx -0{\cdot}6$
and $\quad x \approx 2{\cdot}6$

Alternatively we could simply use a graphic calculator to display the graph of

$$y = 2x^2 - 4x - 3$$

and locate the x-axis intercepts from the display.

This gives solutions of $x = -0{\cdot}58$ (2 dp)
and $x = 2{\cdot}58$ (2 dp)



Completing the square.

Example 4
Solve $x^2 - 4x - 1 = 0$

Given $x^2 - 4x - 1 = 0$

Create the gap: $x^2 - 4x = 1$

Insert the square of half the coefficient of x: $x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 1 + \left(\frac{-4}{2}\right)^2$

Thus
$$\begin{aligned} x^2 - 4x + 4 &= 5 \\ (x-2)^2 &= 5 \\ x - 2 &= \pm\sqrt{5} \\ x &= 2 \pm \sqrt{5} \end{aligned}$$

Giving solutions of 4·24 (2dp) and –0·24 (2dp).

Example 5
Solve $2x^2 + 14x - 5 = 0$

Given $2x^2 + 14x - 5 = 0$

Create the gap (and divide by 2): $x^2 + 7x = \frac{5}{2}$

Insert the square of half the coefficient of x: $x^2 + 7x + \left(\frac{7}{2}\right)^2 = \frac{5}{2} + \left(\frac{7}{2}\right)^2$

Thus
$$\begin{aligned} \left(x + \frac{7}{2}\right)^2 &= 14{\cdot}75 \\ x + 3{\cdot}5 &= \pm\sqrt{14{\cdot}75} \\ x &= -3{\cdot}5 \pm \sqrt{14{\cdot}75} \end{aligned}$$

Giving solutions of 0·34 (2dp) and –7·34 (2dp).

Obtaining and using a formula.

Given $ax^2 + bx + c = 0$

Subtract c from each side and then divide by a: $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Insert the square of half the coefficient of x: $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Thus

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}.$$

∴

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

and so

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 6

Solve $2x^2 + 3x - 1 = 0$

Comparing $2x^2 + 3x - 1 = 0$ with $ax^2 + bx + c = 0$ gives a = 2, b = 3 and c = −1.

Substituting these values into the formula gives $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2}$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

$x = 0{\cdot}28$ (2dp) or $x = -1{\cdot}78$ (2dp)

Note ☞ In the real number system we cannot find the square root of a negative number. Hence if we are attempting to solve a quadratic $ax^2 + bx + c = 0$ for which the quantity $b^2 - 4ac$ is negative it is clear from the quadratic formula that we will have no real solutions.

For example consider the quadratic equation $2x^2 + x + 2 = 0$.

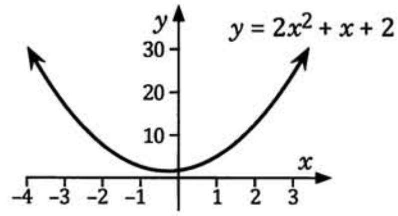
Comparing this equation with $ax^2 + bx + c = 0$ gives a = 2, b = 1 and c = 2.

Thus $\quad b^2 - 4ac = 1^2 - 4 \times 2 \times 2$

$\qquad\qquad\qquad = -15.$

The quadratic equation $2x^2 + x + 2 = 0$ will have no real solutions.

This is also confirmed by the fact that the graph of $y = 2x^2 + x + 2$, shown on the right, does not cut the x-axis. I.e. there are no points on the graph for which $y = 0$.



However, if asked to solve $2x^2 + x + 2 = 0$ a calculator with a quadratic solving program might not say "no solutions" because some branches of mathematics use what are called complex numbers, and the equation does have solutions in this system. However, such work is beyond the requirement of this unit. We only need to know that a calculator response like that shown on the right should be interpreted as *no solutions*, or to be more correct, *no real solutions*. On some calculators a response like that shown can be avoided by setting the calculator to show real solutions only

```
2X²+X+2=0
      X
1 [-0.25+0.9682i]
2 [-0.25-0.9682i]

      -0.25+0.9682458366i
```

☞ The quantity $\mathbf{b^2 - 4ac}$ is called the **discriminant** of the quadratic equation

$$ax^2 + bx + c = 0.$$

It allows us to *discriminate* between (i.e. recognize the distinction between) the three situations of a quadratic having zero, one or two real solutions.

If $b^2 - 4ac > 0$ the quadratic equation $ax^2 + bx + c = 0$ will have two real solutions.

If $b^2 - 4ac = 0$ the quadratic equation $ax^2 + bx + c = 0$ will have one real solutions. (Sometimes referred to as one *repeated* root.)

If $b^2 - 4ac < 0$ the quadratic equation $ax^2 + bx + c = 0$ will have no real solutions.

Exercise 6B

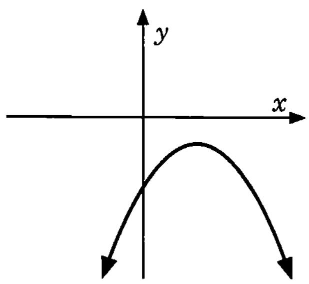
Using the built in facility of some calculators to solve equations.

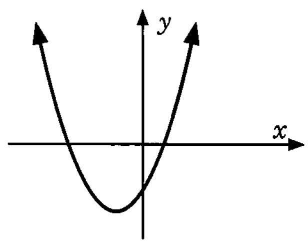
Use a calculator with a built in routine for solving quadratic equations to solve questions 1 to 8, giving your answers correct to 2 decimal places.

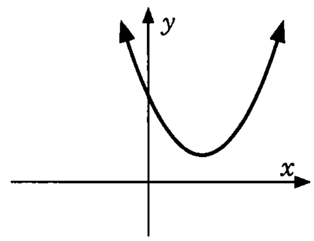
1. $3x^2 + x - 1 = 0$
2. $x^2 + x - 3 = 0$
3. $3x^2 + 3x + 1 = 0$
4. $2x^2 + 6x + 1 = 0$
5. $5x^2 + 7x - 3 = 0$
6. $5x^2 + 6x = 2$
7. $s = ut + \frac{1}{2}at^2$ — Find t given that $s = 35$, $u = -25$, $a = 4$ and $t \geq 0$. (Give your answer correct to one decimal place.)
8. $w = kp^2 - 2cp$ — Find p given that $w = -3$, $k = 5$, $c = 7{\cdot}5$. (Give your answers correct to two decimal places.)

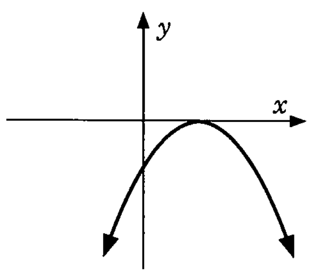
Using a graphical approach.

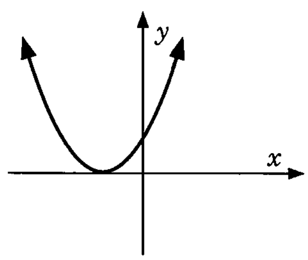
Each of the following graphs are for equations of the form $y = ax^2 + bx + c$.
In each case state the number of real solutions $ax^2 + bx + c = 0$ appears to have.

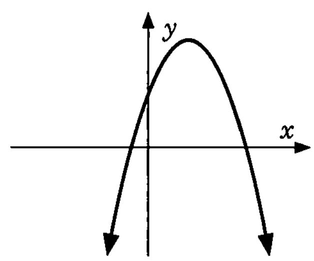
9. 

10. 

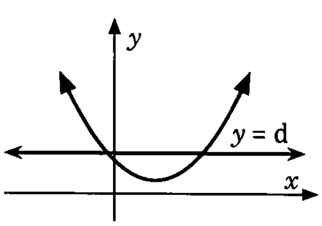
11. 

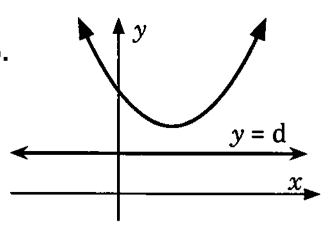
12. 

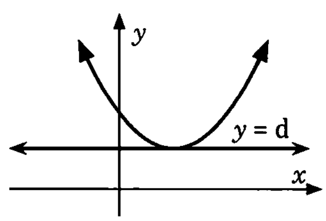
13. 

14. 

Each of the curves below have equations of the form $y = ax^2 + bx + c$.
In each case state the number of real solutions $ax^2 + bx + c = d$ appears to have.

15. 

16. 

17. 

For each of the following first sketch the graph of a suitable quadratic function and then use your sketch to solve the equation.

18. $x^2 + 2x - 2 = 0$	19. $x^2 + 4x - 7 = 0$	20. $2x^2 - 8x + 3 = 0$
21. $x^2 + 2x + 3 = 0$	22. $2x^2 + 12x + 3 = 0$	23. $1 + 4x - x^2 = 0$

Completing the square.

Solve each of the following quadratic equations using the technique of *completing the square*, giving you answers correct to two decimal places if rounding is necessary.

24. $x^2 - 12x + 21 = 0$	25. $x^2 - 6x + 10 = 0$	26. $x^2 - 8x + 1 = 0$
27. $x^2 + 7x - 5 = 0$	28. $x^2 + 3x - 5 = 0$	29. $2x^2 + x - 3 = 0$

Solve each of the following quadratic equations using the technique of *completing the square*, leaving you answers in the form $? \pm \sqrt{?}$, simplified if possible.

30. $x^2 - 2x - 5 = 0$	31. $x^2 - 6x + 1 = 0$	32. $x^2 + 10x - 7 = 0$
33. $2x^2 + 10x - 5 = 0$	34. $3x^2 + 5x + 1 = 0$	35. $5x^2 + x - 1 = 0$

Using the formula.

Solve each of the following quadratic equations using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

giving you answers correct to two decimal places.

36. $x^2 + x - 4 = 0$	37. $7x + 5 - 2x^2 = 0$	38. $3x^2 + 1 = 7x$
39. $6x = x^2 + 7$	40. $x(x - 1) = 7$	41. $2x(3x + 1) = 5$

Solve each of the following quadratic equations using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

leaving you answers in the form $? \pm \sqrt{?}$, simplified if possible.

42. $x^2 + 3x + 1 = 0$	43. $x^2 - 7x + 1 = 0$	44. $2x^2 + x - 5 = 0$
45. $3x^2 = 1 + 5x$	46. $5x^2 - 5 + x = 0$	47. $2x(x + 2) = -1$

By determining the discriminant of each of the following quadratic equations determine the number of real roots each equation has (and then check your answers using the equation solving ability of some calculators).

48. $x^2 + 5x - 7 = 0$	49. $x^2 + 5x + 7 = 0$	50. $x^2 - 2x - 3 = 0$
51. $2x^2 + 7x + 5 = 0$	52. $4x^2 - 12x + 9 = 0$	53. $3x^2 - x + 1 = 0$

Miscellaneous Exercise Six.

This miscellaneous exercise may include questions involving the work of this chapter, the work of any previous chapters, and the ideas mentioned in the preliminary work section at the beginning of the book.

1. The product of 3 less than a number and 5 more than the number is zero. What could the number be?

2. The display on the right shows the lines

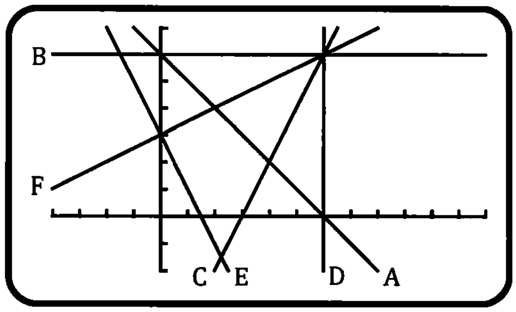
   $x = 60$ $\quad y = 60$

   $y = 2x - 60$ $\quad y = 0{\cdot}5x + 30$

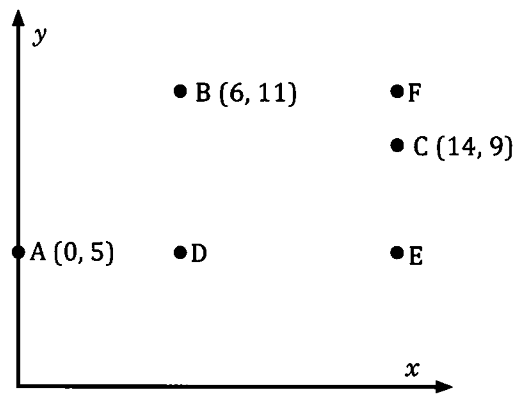
   $y = -x + 60$ $\quad y = -2x + 30$

   labelled A to F.

   Allocate the correct rule to each line.



3. In the diagram on the right the points A, D and E form a horizontal line, as do points B and F. Similarly points E, C and F form a vertical line, as do points D and B. With the coordinates of A, B and C as indicated in the diagram determine:



   (a) The length of AD, the length of DB and hence the gradient of the straight line through A and B.
   (b) The length of DE, the length of EC and hence the gradient of the straight line through D and C.
   (c) The gradient of the straight line through D and F.

4. The graph of the quadratic function $y = 3(x - 1)^2 + 2$ has a line of symmetry with equation $x = a$, has a minimum point at (b, c) and passes through the points $(6, d)$, $(-4, e)$ and $(f, 14)$. Determine a, b, c, d, e and the two possible values of f.

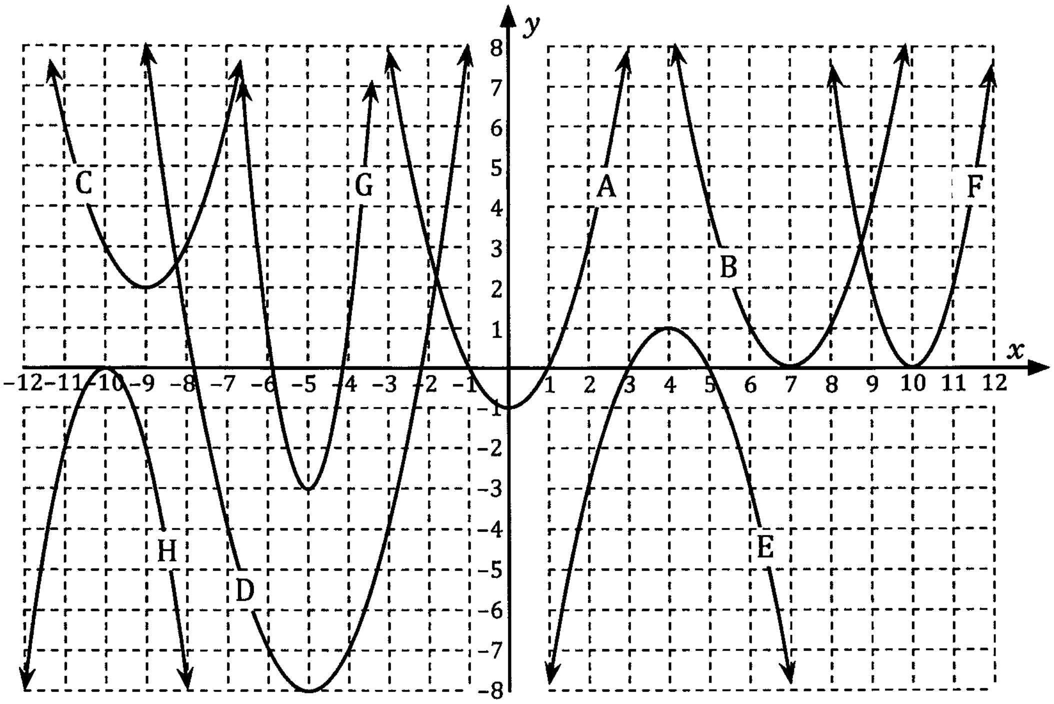
5. A rectangle is such that multiplying the width by five gives an answer that is three centimetres less than the length of the rectangle. If the area of the rectangle is 36 cm^2 find the dimensions of the rectangle.

6. OAB is a right triangle with OA = 5 cm, AB = 10 cm and $\angle$OAB = 90°. A circle of radius 5 cm is drawn, centre O. Find the area of that part of triangle OAB not lying in the circle, giving your answer in square centimetres correct to one decimal place.

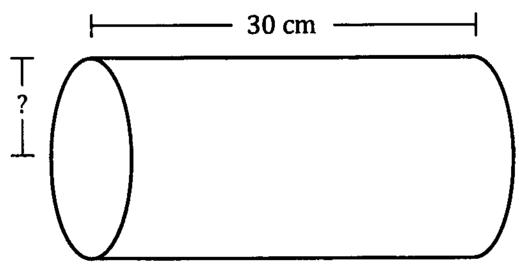
7. Each of the curves A to H shown below are for functions of the form

$$y = a(x - p)^2 + q$$

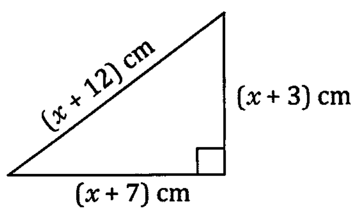
for integer a, p and q.
Determine the equation of each curve.



8. A solid cylinder is to be made having a length of 30 cm and a total surface area of 2000 cm^2.
Determine the radius of such a cylinder, to the nearest millimetre, clearly showing your use of the quadratic formula in your working.



9. Determine the value of x in the right triangle shown sketched on the right, giving your answer rounded to two decimal places and clearly showing your use of the technique of *completing the square* in your working.



10. Apply the technique of completing the square to the general quadratic equation

$$ax^2 + bx + c = 0$$

to obtain the quadratic equation formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$